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Iterative PID Control Design Approach for Maximizing Proportional Gain

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ABSTRACT This paper presents an iterative proportional-integral-derivative (PID) controller design approach. To achieve the desired control performance, the control designer iteratively increases the proportional (P) and derivative (D) gains. And then, the integral controller is added to the obtained PD controller to reduce the steady-state error. Thus, the proposed approach is easy to implement and effective since it aims to maximize P-gain. Moreover, the stability conditions are presented to explain why stability is ensured while increasing PID gains. Simulations for the motor control system are peformed to validate the effectiveness of the proposed approach.

INDEX TERMS control system design, PID control, PID tuning method

I. INTRODUCTION

DUE to the simple structure and intuitive implementation, the proportional-integral-derivative (PID) control technique has been widely utilized in industrial applications such as process control systems [1]–[3], motor control systems [4]–[9], robot manipulators [10]–[12], power systems [13], [14], and magnetic levitation systems [15]. Further, it provides powerful control performance and robustness against model uncertainties and disturbances by merely selecting three tuning parameters. Such wide applications have motivated developing of various PID controller design approaches including Ziegler-Nichols tuning rule [16], [17], Cohen-Coon method [18], internal model controller [19], relay feedback based autotuning [20], [21], pulse response based tuning [22]. In addition, various advanced control techniques have been employed to provide automatic tuning rules. For instance, the data-driven PID design [23], linear matrix inequality (LMI) based optimization method [24], [25], generic algorithm [26], [27], and reinforcement learning-based tuning method [28] have been presented.

PID controller design approaches mentioned above are roughly classified into model-based and model-free methods [29]. Model-based methods require accurate information about the plant model and sometimes assume that the plant is modeled as the first-order plus time delay or second-order plus time delay systems. Model-free methods are based on input-output data obtained by experiments and the minimization of certain objective functions. However, in real situations, various inevitable factors like plant nonlinearities and model uncertainties, external disturbances, and load variations lead to control performance degradation. Even within a small operating region, selecting PID gains to achieve the desired transient and steady-state performance in the region of interest is a challenging task. Thus, after an initial selection of PID gains by the approaches above, fine-tuning of PID gains is required through a trial-and-error procedure [16].

Indeed, each parameter of PID controller has its own physical interpretation in terms of the performance of the closed-loop system [30]. The proportional (P) gain provides a control action, proportional to the error between the desired and actual outputs, to follow the output to the desired reference. The closed-loop system response improves as P-gain increases. The integral (I) gain represents the accumulated control effort using past error information. Adding an integrator into the control loop reduces the steady-state error. However, increasing I-gain may degrade the stability of the closed-loop system. The derivative (D) gain implies the anticipated control effort reflecting the rate of change of the ongoing error. Thus, the transient response, such as rise time, overshoot, and settling time, is improved by increasing D-gain. Based on the physical meaning of each PID term and output response, the control designer can iteratively tune PID parameters to satisfy the desired control objectives. However, it is a time-consuming procedure and requires an expert control engineer to obtain satisfactory control performance.

One of the simple but effective tuning methods is to maximize P-gain by alternately increasing P- and D-gains repeatedly. First, the control designer increases P-gain until the system oscillates. Then, D-gain is increased to mitigate the oscillation to improve the transient response. In a sequential manner, the designer increases P- and D-gains to attain the desired control performance iteratively. Finally, if necessary, I controller is added to the obtained PD controller to reduce the steady-state error. Since it is easy to implement and improve control performance, this approach has been utilized in tuning the motor controllers for mechanical systems with a large number of degrees of freedom (e.g., humanoid and articulated robots). Nonetheless, to the best of the authors' knowledge, there is no detailed design procedure or theoretical analysis for stability and performance improvement.

This paper deals with the iterative PID tuning approach to maximize P-gain. The main contributions and theoretical developments are as follows. We rigorously formulate the iterative PID tuning procedure and explain how the proposed approach enhances the control performance by maximizing P-gain. Then, the stability conditions are presented to explain why stability is guaranteed while increasing PID gains. The presented analysis explains why the high gain control may destabilize the closed-loop system in real applications. Furthermore, to validate the effectiveness of the proposed approach, simulations for the motor control system are performed.

The rest of this paper is organized as follows. In the following section, the preliminary results for *Hurwitz* stability based on the interlacing property and the problem under consideration are introduced. Section III proposes the iterative PID design procedure and discusses the stability of the closed-loop system while increasing PID gains. In Section IV, simulations for the motor control system are performed to verify the proposed approach. Finally, we conclude this paper in Section V.

II. PROBLEM FORMULATION

Routh-Hurwitz stability criterion is a useful tool for investigating the stability of linear time-invariant systems. In this paper, we introduce an alternative stability condition for linear systems based on interlacing property. In addition, the PID control design problem under consideration is formulated.

A. PRELIMINARIES

Consider the following polynomial with real coefficients a_i.

$$\mathbf{g}(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0. \tag{1}$$

The polynomial $\mathbf{g}(s)$ is said to be of degree n if $a_n \neq 0$. A *Hurwitz* polynomial implies that all its roots are in the open left half complex plane. The odd and even part polynomials of (1) are denoted by

$$\begin{aligned} \mathbf{g}_{even}(s) &= a_0 + a_2 s^2 + a_4 s^4 + \cdots, \\ \mathbf{g}_{odd}(s) &= a_1 s + a_3 s^3 + a_5 s^5 + \cdots. \end{aligned} \tag{2}$$

In addition, we define

$$\begin{aligned} \mathbf{g}_e(\omega) &= \mathbf{g}_{even}(j\omega) = a_0 - a_2\omega^2 + a_4\omega^4 + \cdots, \\ \mathbf{g}_o(\omega) &= \mathbf{g}_{odd}(j\omega)/j\omega = a_1 - a_3\omega^2 + a_5\omega^4 + \cdots. \end{aligned} \tag{3}$$

Definition 1: The polyomial $\mathbf{g}(s)$ of degree n satisfies the interlacing property if

a) The degree of $\mathbf{g}(s)$ is even (i.e., $n = 2k$) with

$$\begin{aligned} \mathbf{g}_e(\omega) &= (-1)^k a_{2k}\omega^{2k} + \cdots - a_2\omega^2 + a_0, \\ \mathbf{g}_o(\omega) &= (-1)^{k-1} a_{2k-1}\omega^{2k-2} + \cdots - a_3\omega^2 + a_1. \end{aligned}$$

b) The coefficients a_{2k} and a_{2k-1} have the same sign.

c) All the roots of $\mathbf{g}_e(\omega)$ and $\mathbf{g}_o(\omega)$ are real and distinct, and the k positive roots of $\mathbf{g}_e(\omega)$ together with the $k-1$ positive roots of $\mathbf{g}_o(\omega)$ interlace as follows:

$$0 < \omega_{e,1} < \omega_{o,1} < \cdots < \omega_{o,k-1} < \omega_{e,k}$$

or if

a) The degree of $\mathbf{g}(s)$ is odd (*i.e.*, $n = 2k+1$) with

$$\begin{aligned} \mathbf{g}_e(\omega) &= (-1)^k a_{2k}\omega^{2k} + \cdots - a_2\omega^2 + a_0, \\ \mathbf{g}_o(\omega) &= (-1)^k a_{2k+1}\omega^{2k} + \cdots - a_3\omega^2 + a_1. \end{aligned}$$

b) The coefficients a_{2k+1} and a_{2k} have the same sign.

c) All the roots of $\mathbf{g}_e(\omega)$ and $\mathbf{g}_o(\omega)$ are real and distinct, and the k positive roots of $\mathbf{g}_e(\omega)$ together with the k positive roots of $\mathbf{g}_o(\omega)$ interlace as follows:

$$0 < \omega_{e,1} < \omega_{o,1} < \cdots < \omega_{e,k} < \omega_{o,k}$$

□

Theorem 1: The polynomial $\mathbf{g}(s)$ is a *Hurwitz* polynomial if and only if it satisfies the interlacing property. □

For more details on the interlacing property (Definition 1 and Theorem 1), please refer to [31].

We provide the following two examples to illustrate Theorem 1.

Example 1: Consider a third-order polynomial $\mathbf{g}^3(s) = s^3 + 3s^2 + 3s + 1$. Then, we have

$$\begin{aligned} \mathbf{g}_e^3(\omega) &= -3\omega^2 + 1, \\ \mathbf{g}_o^3(\omega) &= -\omega^2 + 3. \end{aligned} \tag{4}$$

The positive roots of $\mathbf{g}_e^3$ and $\mathbf{g}_o^3$ are $\omega_{e,1}^3 = 1/\sqrt{3}$ and $\omega_{o,1}^3 = \sqrt{3}$. Thus, $\mathbf{g}^3(s)$ satisfies the interlacing property and obviously it is a *Hurwitz* polynomial. □

Example 2: Consider a sixth-order polynomial

$$\mathbf{g}^6(s) = s^6 + 6s^5 + 15s^4 + 20s^3 + 15s^2 + 6s + 1. \tag{5}$$

The even and odd parts of (5) are given by

$$\begin{aligned} \mathbf{g}_e^6(\omega) &= -\omega^6 + 15\omega^4 - 15\omega^2 + 1, \\ \mathbf{g}_o^6(\omega) &= 6\omega^4 - 20\omega^2 + 6. \end{aligned}$$

Then, as shown in Fig. 1, the positive roots of $\mathbf{g}_e^6(\omega)$ and $\mathbf{g}_o^6(\omega)$ are $\omega_{e,1}^6 = 2 - \sqrt{3} \approx 0.2679$, $\omega_{e,2}^6 = 1$, and $\omega_{e,3}^6 = 2 + \sqrt{3} \approx 3.7321$ and $\omega_{o,1}^6 = \sqrt{3}/3 \approx 0.5774$ and $\omega_{o,2}^6 = \sqrt{3} \approx 1.7321$, respectively. Thus, the interlacing property holds and $\mathbf{g}^6(s)$ is a *Hurwitz* polynomial. □

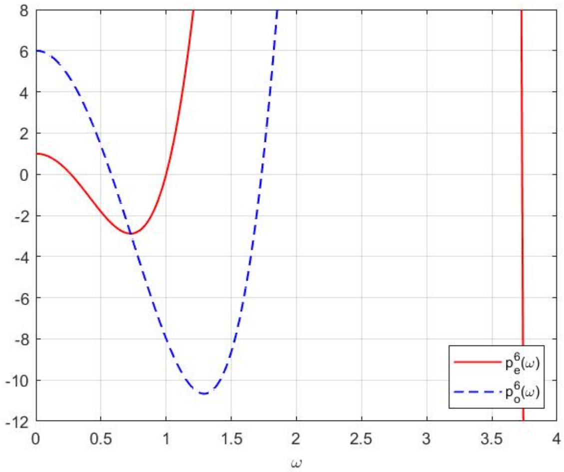

FIGURE 1. Even and odd parts of polynomial $p^6(s)$ in (5)

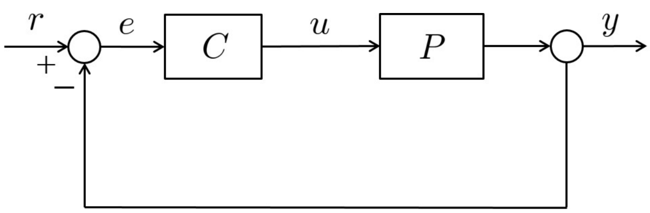

FIGURE 2. Configuration of the closed-loop system

B. CONFIGURATION OF CLOSED-LOOP SYSTEM WITH PROPORTIONAL-INTEGRAL-DERIVATIVE CONTROL

Consider the feedback control system in Fig. 2. The actual plant P is of the form.

$$P(s) = \frac{\mathbf{n}(s)}{\mathbf{d}(s)} = \frac{1}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}. \tag{6}$$

where a_i's are real coefficients and the positive integer n is the degree of the denominator.

To stabilize P and satisfy the control objectives such as rise time, overshoot, settling time, and steady-state error, we employ PID controller C as follows:

$$C(s) = K_P + K_I\frac{1}{s} + K_D s. \tag{7}$$

where K_P, K_I, and K_D are the proportional, integral, and derivative control gains, respectively. Depending on specific applications, different variants of PID controllers are utilized. For example, the proportional-derivative (PD) controller (i.e., $K_I = 0$) has been widely employed in robot manipulator systems [32]. In contrast, the proportional-integral (PI) controller (i.e., $K_D = 0$) has been used in motor control systems [33]. Here, the signals r, y, e, and u stand for the reference, output, error, and control input, respectively. The transfer function from the reference input to the output is represented as

$$\frac{y(s)}{r(s)} = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{\mathbf{n}_{cl}(s)}{\mathbf{d}_{cl}(s)}. \tag{8}$$

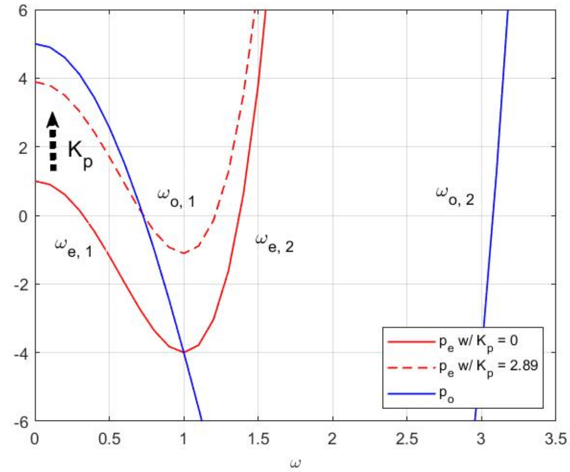

FIGURE 3. Even and odd parts of characteristic polynomial with $K_P = 0$ and $K_P = 2.89$

Definition 2: The closed-loop system (8) is said to be *Hurwitz* stable if and only if $\mathbf{d}_{cl}(s)$ is a *Hurwitz* polynomial. □

Now, we discuss how to design a PID controller for achieving the desired performance by maximizing K_P and guaranteeing the stability of the overall closed-loop system.

III. ITERATIVE DESIGN APPROACH FOR PROPORTIONAL-INTEGRAL-DERIVATIVE (PID) CONTROL

A. MOTIVATING EXAMPLE

This subsection presents an example how to select PD control gains iteratively for maximizing P-gain. Consider a fifth-order plant given by

$$P(s) = \frac{1}{s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + 1}.$$

Initially, we design a proportional (P) controller $C(s) = K_P$. The characteristic polynomial of the closed-loop system and its even and odd parts are given by

$$\begin{aligned}\mathbf{g}^1(s) &= s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + 1 + K_P,\\ \mathbf{g}_e^1(\omega) &= 5\omega^4 - 10\omega^2 + 1 + K_P,\\ \mathbf{g}_o^1(\omega) &= \omega^4 - 10\omega^2 + 5.\end{aligned}$$

Fig. 3 shows the plot of $\mathbf{g}_e^1(\omega)$ and $\mathbf{g}_o^1(\omega)$ with respect to K_P. As K_P increases, the magnitude of $\mathbf{g}_e^1(\omega)$ grows and $\omega_{e,1}$, the smallest positive root of $\mathbf{g}_e^1(\omega)$, moves to the right to $\omega_{o,1}$, the smallest positive root of $\mathbf{g}_o^1(\omega)$, until K_P reaches 2.89. Thus, the closed-loop system is *Hurwitz* stable with $0 \le K_P < 2.89$.

Next, we choose the derivative gain K_D for PD controller $C(s) = K_P + K_D s$ for a fixed $K_P = 2.89$. Then, we have

$$\begin{aligned}\mathbf{g}^2(s) &= s^5 + 5s^4 + 10s^3 + 10s^2 + (5 + K_D)s + 3.89,\\ \mathbf{g}_e^2(\omega) &= 5\omega^4 - 10\omega^2 + 3.89,\\ \mathbf{g}_o^2(\omega) &= \omega^4 - 10\omega^2 + 5 + K_D.\end{aligned}$$

As shown in Fig. 4, $\omega_{o,1}$ moves to the right from $\omega_{e,1}$ to $\omega_{e,2}$ as K_D increases. Then, the interlacing property is preserved and

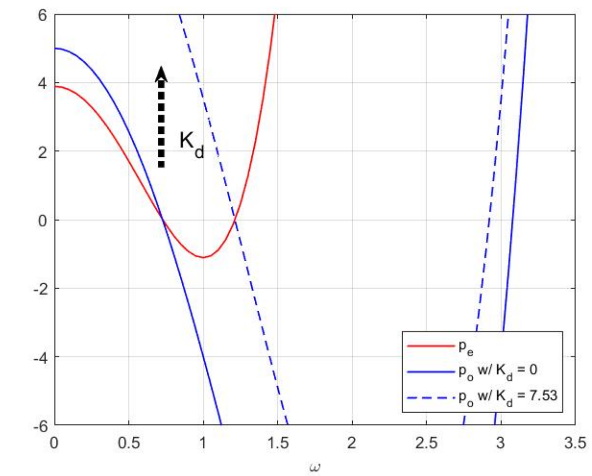

FIGURE 4. Even and odd parts of characteristic polynomial with $K_P = 2.89$, and $K_D = 0$ and $K_D = 7.53$

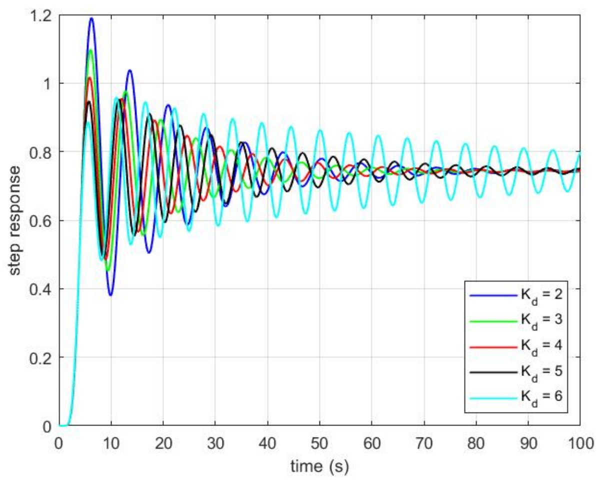

FIGURE 5. Step responses of the closed-loop system with respect to K_D

the closed-loop system is stable for $0 < K_D < 7.53$. Since the gain K_D affects the transient response and the excessively large value makes the system oscillate, to achieve the desired behavior, K_D is determined using the trial-and-error method within the stable region. Fig. 5 presents the step responses of the closed-loop system with respect to K_D. To reduce the overshoot and oscillation simultaneously, we select $K_D = 3$. In the next step, we obtain $K_P = 3.92$ and $K_D = 5$. Similarly, we use the iterative procedure to increase the proportional gain.

B. STABILITY CONDITION FOR INCREASING PID GAINS

In this subsection, we deal with why stability is guaranteed despite increasing the gains of PID controller. The first two theorems are for increasing P- and D-control gains in an iterative PD control design procedure. After selecting the PD gains, the last theorem shows how to select I-gain K_I to improve the disturbance rejection performance and reduce the steady-state error of the closed-loop system.

Theorem 2: Suppose that the plant $P(s)$ in (6) is *Hurwitz* stable. Then, there exists a positive constant $\overline{K}_P$ such that, for all $0 \leq K_P < \overline{K}_P$, the closed-loop system with P controller $C(s) = K_P$ in Fig. 2 is *Hurwitz* stable. □

Proof: If $n \leq 2$, then the proof is trivial since the first or second-order polynomial having positive coefficients is always *Hurwitz*. Thus, for any $\overline{K}_P > 0$, the closed-loop system with a P controller is *Hurwitz* stable for all $0 \leq K_P < \overline{K}_P$.

The characteristic polynomial of the closed-loop system is computed as

$$\mathbf{p}(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 + K_P. \quad (9)$$

We first consider the case that the degree of (9) is odd (i.e., $n = 2k + 1$ where k is a positive integer). Then, the odd and even parts of (9) are represented as

$$\begin{aligned} \mathbf{g}_e(\omega) &= (-1)^k a_{2k}\omega^{2k} + \cdots - a_2\omega^2 + (a_0 + K_P), \\ \mathbf{g}_o(\omega) &= (-1)^k a_{2k+1}\omega^{2k} + \cdots - a_3\omega^2 + a_1. \end{aligned} \quad (10)$$

The plant is *Hurwitz* stable, so the characteristic polynomial in (9) with $K_P = 0$ satisfies the interlacing property:

$$0 < \omega_{e,1} < \omega_{o,1} < \cdots < \omega_{e,k} < \omega_{o,k}. \quad (11)$$

where $\omega_{e,i}$ and $\omega_{o,i}$ are the positive roots of $\mathbf{g}_e(\omega)$ and $\mathbf{g}_o(\omega)$, respectively. As K_P increases, the function $\mathbf{g}_e(\omega)$ goes up. Hence, for a positive integer i, $\omega_{e,2i-1}$ moves to the right to $\omega_{o,2i-1}$ and $\omega_{e,2i}$ moves to the left to $\omega_{o,2i-1}$. Thus, there exists a positive constant $\overline{K}_P$ such that the interlacing property does not hold (i.e., $\omega_{e,2i-1}$ or $\omega_{e,2i}$ intersect $\omega_{o,2i-1}$).

Now, let us suppose that the degree of (9) is even (i.e., $n = 2k + 2$) and we have

$$\begin{aligned} \mathbf{g}_e(\omega) &= (-1)^{k+1} a_{2k+2}\omega^{2k+2} + \cdots - a_2\omega^2 + (a_0 + K_P), \\ \mathbf{g}_o(\omega) &= (-1)^k a_{2k+1}\omega^{2k} - \cdots - a_3\omega^2 + a_1. \end{aligned} \quad (12)$$

When $K_P = 0$, the positive roots of $\mathbf{g}_e(\omega)$ and $\mathbf{g}_o(\omega)$ also satisfy the following inequality:

$$0 < \omega_{e,1} < \omega_{o,1} < \cdots < \omega_{o,k} < \omega_{e,k+1}. \quad (13)$$

As K_P increases, for a positive integer i, $\omega_{e,2i-1}$ moves to the right to $\omega_{o,2i-1}$ while $\omega_{e,2i}$ moves to the left to $\omega_{o,2i-1}$. The last root $\omega_{e,k+1}$ moves to the right and does not intersect the root of $\mathbf{g}_o(\omega)$. Therefore, there exists a positive constant $\overline{K}_P$ such that, for all $0 \leq K_P < \overline{K}_P$, the inequality (13) holds and it conclude the proof. ■

The following lemma presents how stability is preserved while increasing P-gain.

Lemma 3: Suppose that PD control gains K_P and K_D are designed for the plant (6) such that the closed-loop system with PD controller $C(s) = K_P + K_D s$ in Fig 2 is *Hurwitz* stable. Then, there exists a positive constant $\overline{K}_P$ such that, for all $K_P < \tilde{K}_P < \overline{K}_P$, the closed-loop system with PD controller $C(s) = \tilde{K}_P + K_D s$ in Fig. 2 is *Hurwitz* stable. □

Proof: We assume that PD controller is designed to stabilize the plant (6), so its characteristic polynomial

$$\mathbf{p}_{pd}(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + (a_1 + K_D) + (a_0 + K_P) \quad (14)$$

is *Hurwitz*. Then, the characteristic polynomial of the closed-loop system with PD controller is represented as

$$\begin{aligned}\tilde{\mathbf{p}}_{pd}(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + (a_1 + K_D)s \\ + (a_0 + K_P + \tilde{K}_P - K_P)\end{aligned} \quad (15)$$

Since $0 < \tilde{K}_P - K_P < \overline{K}_P - K_P$, from Theorem 2, (15) is the *Hurwitz* polynomial, which concludes the proof. ■

As a counterpart of Theorem 2 and Lemma 3, we provide results for designing D-gain.

Theorem 4: Suppose that the plant $P(s)$ in (6) is *Hurwitz* stable. Then, there exists a positive constant $\overline{K}_D$ such that, for all $0 \leq K_D < \overline{K}_D$, the closed-loop system with D controller $C(s) = K_D s$ in Fig. 2 is *Hurwitz* stable. □

Proof: Since the proof for the case $n \leq 2$ is trivial, we start the proof from $n \geq 3$. The characteristic polynomial of the closed-loop system is

$$\mathbf{p}(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + (a_1 + K_D)s + a_0. \quad (16)$$

The odd and even parts of (16) are obtained as follows: if the degree of (16) is odd, then

$$\begin{aligned}\mathbf{g}_e(\omega) &= (-1)^k a_{2k}\omega^{2k} + \cdots - a_2\omega^2 + a_0, \\ \mathbf{g}_o(\omega) &= (-1)^k a_{2k+1}\omega^{2k} + \cdots - a_3\omega^2 + (a_1 + K_D),\end{aligned} \quad (17)$$

on the other hand, if the degree of (16) is even, then

$$\begin{aligned}\mathbf{g}_e(\omega) &= (-1)^{k+1} a_{2k+2}\omega^{2k+2} + \cdots - a_2\omega^2 + a_0, \\ \mathbf{g}_o(\omega) &= (-1)^k a_{2k+1}\omega^{2k} - \cdots - a_3\omega^2 + (a_1 + K_D).\end{aligned} \quad (18)$$

As shown in (17) and (18), K_D only affects the function $g_o(\omega)$ and the value of $g_o(\omega)$ increases as K_D increases. The rest of the proof is similar to that of Theorem 2. Thus, we omit the details. ■

Lemma 5: Suppose that PD control gains K_P and K_D are designed for the plant (6) such that the closed-loop system with PD controller $C(s) = K_P + K_D s$ in Fig 2 is *Hurwitz* stable. Then, there exists a positive constant $\overline{K}_D$ such that, for all $K_D < \tilde{K}_D < \overline{K}_D$, the closed-loop system with PD controller $C(s) = K_P + \tilde{K}_D s$ in Fig. 2 is *Hurwitz* stable. □

Proof: The detailed proof is almost the same as that of Lemma 3 except that the characteristic polynomial of the closed-loop system is $a_n s^n + a_{n-1}s^{n-1} + \cdots + (a_1 + \tilde{K}_D)s + a_0 + K_P$. Hence, we omit it here. ■

Note that Theorems 2 and 4 are only applicable to *Hurwitz* stable systems. Even though they seem restrictive, the systems considered can be easily extended to a more general class of systems. For instance, if plant (6) is input feedforward passive (i.e., the plant is marginally stable except in some special cases where the poles are in $j\omega$-axis), the closed-loop system is asymptotically stabilizable by a simple static output feedback control (e.g., P control) [34], [35]. Thus, the above results are applicable to motor and general mechanical systems having a single integrator.

The next theorem is for adding I-control to a pre-designed PD controller.

Theorem 6: Suppose that PD control gains K_P and K_D are designed for plant (6) such that the closed-loop system with PD controller $C(s) = K_P + K_D s$ in Fig 2 is *Hurwitz* stable. Then, there exists a positive constant $\overline{K}_I$ such that, for all $0 \leq K_I < \overline{K}_I$, the closed-loop system with PID controller $C(s) = K_P + K_I(1/s) + K_D s$ in Fig. 2 is *Hurwitz* stable. □

Proof: By the assumption for PD control, the characteristic polynomial

$$\mathbf{p}_{pd}(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + (a_1 + K_D) + (a_0 + K_P) \quad (19)$$

is *Hurwitz*. Then, the characteristic polynomial with PID controller is calculated as

$$\begin{aligned}\mathbf{p}(s) &= a_n s^{n+1} + a_{n-1}s^n + \cdots \\ &\quad + (a_1 + K_D)s^2 + (a_0 + K_P)s + K_I \\ &= s\mathbf{p}_{pd}(s) + K_I\end{aligned} \quad (20)$$

In fact, the roots of $\mathbf{p}(s)$ are the poles of the unity feedback system with the transfer function $H(s) = K_I/(s\mathbf{p}_{pd}(s))$. Recall that, from the root locus method, the loci are on the real axis to the left of an odd number of poles and zeros [36]. $H(s)$ has no zeros and all the poles are in the left half complex plane except for one pole at the origin. It implies that this pole at the origin moves to the left on the negative real axis while the other poles stay in the left half complex plane as K_I increases from zero [31], [37]. Thus, for a sufficiently small K_I, the closed-loop system is *Hurwitz* stable. This concludes the proof. ■

C. ITERATIVE DESIGN APPROACH FOR PROPORTIONAL-INTEGRAL-DERIVATIVE (PID) CONTROL

Inspired by the motivating example discussed in subsection III-A, we present an iterative design procedure for PID control to maximize K_P while guaranteeing stability.

Iterative Design Procedure for PID Control

Step 0*:* Set initial P-gain K_P^0 and D-gain K_D^0. Select a tolerance $\epsilon > 0$ and a maximum number of iterations m.

Step 1*:* Increase P-gain from K_P^0, find $\overline{K}_P^0$ when the output of the closed-loop system exhibits sustained oscillations (i.e., the closed-loop system is marginally stable), and select K_P^1 such that $K_P^0 < K_P^1 < \overline{K}_P^0$. Then, for the chosen P-gain K_P^1, increase D-gain from K_D^0 and select K_D^1 such that the output of the closed-loop system meets the desired transient response (if possible, it is critically damped).

Step i: Increase P-gain from K_P^{i-1}, find $\overline{K}_P^{i-1}$ when the output exhibits sustained oscillations, and select K_P^i such that $K_P^{i-1} < K_P^i < \overline{K}_P^{i-1}$. Then, for the chosen P-gain K_P^i, increase D-gain from K_D^{i-1} and select K_D^i such that the output meets the desired transient response. If $K_P^i - K_P^{i-1} < \epsilon$, then set $K_p = K_P^i$ and $K_D = K_D^i$ and go to *Step m*.

Step m − 1*:* Increase P-gain from K_P^{m-2}, find $\overline{K}_P^{m-2}$ when the output exhibits sustained oscillations, and select K_P^{m-1}

such that $K_P^{m-2} < K_P^{m-1} < \overline{K}_P^{m-2}$. Then, for the chosen K_P^{m-1}, increase D-gain from K_D^{m-2} and select K_D^{m-1} such that the output meets the desired transient response. Set $K_P = K_P^{m-1}$ and $K_D = K_D^{m-1}$ and go to *Step m*.

Step m: If necessary, with the obtained K_P and K_D, increase I-gain from 0 and select an appropriate K_I to reduce the steady-state error while guaranteeing stability. Stop the procedure.

It is an iterative procedure. *Step* 0 is the initialization step. An appropriate PID control design method chooses K_P^0 and K_D^0 such that the closed-loop system with PD control is *Hurwitz* stable. Since a large P-gain enhances the control system performance, the designer may try to continuously repeat the tuning process to obtain as large P-gain possible. However, it is time-consuming work and a trade-off between cost and performance needs to be balanced. Therefore, two design parameters ϵ and m are determined taking into account such practical implementation.

Let us move to *Step* 1. By Lemma 3, the closed-loop system remains *Hurwitz* stable while increasing P-gain from K_P^0 to $\overline{K}_P^0$. Then, for the chosen K_P^1, the closed-loop system remains *Hurwitz* stable while increasing D-gain by Lemma 5. In a similar manner, the proposed procedure proceeds sequentially to *Step m*. Therefore, as the step goes on, K_P^i and K_D^i gradually grow and are monotonically increasing sequences. When $n \leq 3$, the designer can select an arbitrarily large K_P and K_D using the proposed algorithm. When $n \leq 2$, then selecting large gain parameters is trivial since the closed-loop system is *Hurwitz* stable for all $K_P > 0$ and $K_D > 0$. When $n = 3$, each function $\mathbf{g}_e(\omega)$ and $\mathbf{g}_o(\omega)$ has only one positive root $\omega_{e,1}$ and $\omega_{o,1}$, respectively. It implies that there is no root of $\mathbf{g}_e(\omega)$ on the right side of $\omega_{o,1}$. Thus, K_D can be increased unboundedly and, for any arbitrarily large K_P, one can find K_D by the proposed iterative procedure such that the closed-loop system is *Hurwitz* stable. However, in practical applications, the gains are limited by the effects of system nonlinearities and input saturation. In contrast, when $n \geq 4$, there is an upper bound $K_P^\star$ for a monotonically increasing sequence K_P^i. As K_P increases, $\mathbf{g}_e(\omega)$ of (9) goes up, and any two roots $\omega_{e,2i-1}$ and $\omega_{e,2i}$ get closer and eventually merge with each other (i.e., $\omega_{e,2i-1}$ moves to the right and $\omega_{e,2i}$ moves to the left). Hence, $\omega_{e,2i-1}$ and $\omega_{e,2i}$ become repeated roots and the interlacing property does not hold. These observations explain why the high gain may destabilize the closed-loop system in the actual controller design.

In robot manipulator systems, PD control is sufficient to meet the desired control objectives [32]. However, if there is a steady-state error, then I control is a remedy for this problem. When it is necessary, I controller is added into the predetermined PD controller at *Step m*. As discussed in Theorem 6, for small K_I, the poles of the closed-loop system with PID control stay around the poles of the closed-loop system with PD control (i.e., $K_I = 0$) except the one pole near the origin. Thus, a simple design guideline for I-gain is that the designer selects a small K_I to reduce the steady-state error and simultaneously maintain the pre-designed transient response.

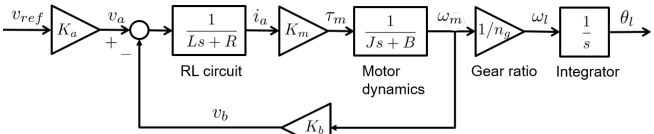

FIGURE 6. Diagram of Electromechanical System

TABLE 1. Parameters of two-inertia system

J_m	Motor inertia	0.00844 $lb_f - in - s^2$
J_l	Load inertia	1 $lb_f - in - s^2$
B_m	Motor shaft viscous damping coefficient	0.00013 $lb_f - in/deg/s$
B_l	Load shaft viscous damping coefficient	0.5 $lb_f - in/deg/s$
L	Armature inductance	0.0006 H
R	Armature resistance	1.4 Ω
K_a	Amplifier gain	12
K_b	Back emf constant	0.00867$V/deg/s$
K_m	Torque constant	4.375$lb_f - in/A$
n_g	Gear ratio	200

IV. APPLICATION TO MOTOR CONTROL SYSTEM

In order to validate the proposed PID design approach, we design a PID controller for an electromechanical system in Fig. 6. The variables θ_m and ω_m are the motor shaft angle and velocity, respectively, θ_l and ω_l are the load shaft angle and velocity, respectively, τ_m is the generated motor torque, i_a and v_a are the armature current and voltage, respectively, v_b and v_{ref} are the back emf and reference voltages, respectively. The values of each parameter J_m, J_l, B_m, B_l, L, R, K_a, K_b, K_m, and n_g are given in Table 1 [38]. The transfer function from v_{ref} to θ_l is represented as

$$\mathbf{P}(s) = \frac{1}{a_{p3}s^3 + a_{p2}s^2 + a_{p1}s} \tag{21}$$

where $a_{p1} = n_g(BR + K_b)/K_a$, $a_{p2} = n_g(JR + BL)/K_a$, and $a_{p3} = n_gJL/K_a$. Here, $J = J_m + J_l/n_g^2$ and $B = B_m + B_l/n_g^2$ are the effective inertia and damping coefficients, respectively.

Using the proposed iterative PID design approach, we designed PD gains with $m = 3$ as follows. At *Step 0*, we set $K_P^0 = 10$ and $K_D^0 = 3$ from initial guessing. Then, at *Step 1*, the output of the closed-loop system exhibits sustained oscillation with $\overline{K}_P^0 = 456$. We select $K_P^1 = 450$ and $K_D^1 = 15.5$ to make the step response of the closed-loop system critically damped. Iteratively, we choose $K_P^2 = 30000$ and $K_D^2 = 122$ at *Step 2*. At *Step 3*, PD gains are selected as $K_P^3 = 255000$ and $K_D^3 = 230$ to prevent excessive overshoot. Note that the simulations are performed in Simulink/Matlab environment using ode45 method. Fig. 7 shows the step responses of the closed-loop system with PD control gains at each step. Since the output follows the reference input without steady-state error, I control is unnecessary. Thus, we complete the iterative procedure at *Step 3*. It is observed that, as the step proceeds, the performance of the closed-loop system, such as rise time and system response, improves. Thus, the control designer

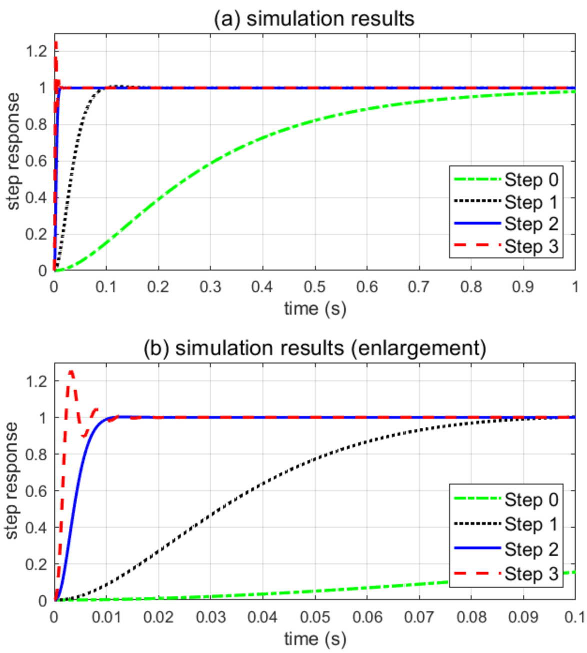

FIGURE 7. **Step responses of the motor control system with PD control designed using the proposed algorithm**

TABLE 2. **Control gains of three PID controllers**

Tuning approach	K_P	K_D	K_I
Proposed approach	200, 000	230	–
Ziegler-Nichols (ZN)	204	3.825	2720
Phase margin (PM)	170	7.65	878.553

can easily design a PID controller using the iterative gain tuning approach.

To further show the efficacy of the proposed iterative PID tuning approach, we compare the step responses of the closed-loop system with PID controllers designed by the proposed approach, Ziegler-Nichols method (ZN) [16], and a phase margin design rule (PM) (the desired phase margin is 60^o) [39]. Detailed control gains of each PID controller are listed in Table 2. It can be observed that K_p and K_d gains of the proposed approach are much larger than those of the other two approaches. Therefore, as depicted in Fig. 8, the performance of the proposed approach outperforms those of the other two approaches, despite lacking the integral control term.

V. CONCLUSION

In this paper, we propose an iterative PID controller design approach that maximizes P-gain to improve the control performance. The proposed approach is simple and easy to implement since it iteratively increases P- and D-gains. After selecting PD gains, I controller is added to the obtained PD controller to reduce the steady-state error. To rigorously explain the proposed approach, the stability conditions are presented to explain why stability is guaranteed while increasing PID gains. To verify the performance of the proposed approach, we implement it for a motor control system.

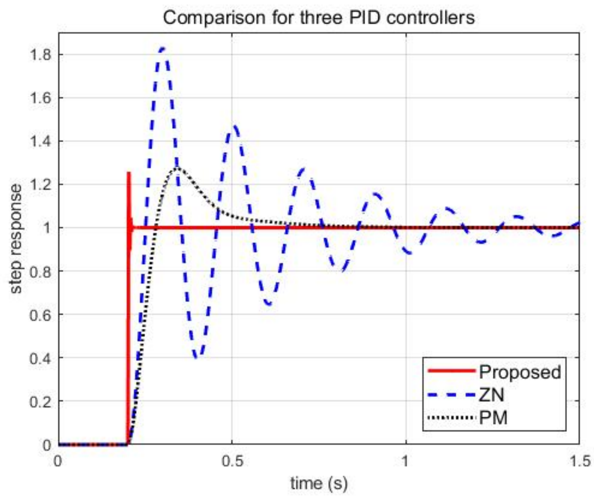

FIGURE 8. **Step responses of the motor control system with PID controllers designed by the proposed approach (Proposed, solid), Ziegler-Nichols method (ZN, dashed), and phase margin design rule (PM, dotted)**

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